GESTÃO, FINANÇAS E CONTABILIDADE

LINEAR PROGRAMMING APPLIED TO FINANCE - BUILDING A GREAT PORTFOLIO INVESTMENT

PROGRAMAÇÃO LINEAR APLICADA A FINANÇAS – CONSTRUINDO UM PORTFÓLIO ÓTIMO

PROGRAMACION LINEAL APLICADOS A LA FINANZAS - GRAN EDIFICIO A LA CARTERA

Pedro Marinho Sizenando Silva
Doutorando em Engenharia de Produção (UFMG)
Professor do Instituto Federal de Minas Gerais - Campus Formiga
Endereço: Rua Padre Alberico, s/n - São Luiz
35.570-000 – Formiga/MG, Brasil
Email: pedro.silva@ifmg.edu.br

Bruno César Melo Moreira
Doutor em Administração (UFSC)
Professor do Instituto Federal de Minas Gerais - Campus Formiga
Endereço: Rua Padre Alberico, s/n - São Luiz
35.570-000 – Formiga/MG, Brasil
Email: bruno.melo@ifmg.edu.br

Gleison de Almeida Francisco
Tecnólogo em Gestão Financeira (IFMG)
Email: gleison.a.francisco@hotmail.com

ABSTRACT
The stock market has grown steadily in recent years, and several indices have also been created in this market, like IGC, ISE and IBOVESPA. Thinking about this market growth, this paper aims to build an optimal portfolio using linear programming, based on companies simultaneously present in the indices: IGC and ISE. The constraints of the problem will be based on indicators of IBOVESPA. The model will be created to meet the restrictions set and to maximize the portfolio return, always comparing with the return of IBOVESPA, with a time horizon from 2007 until 2012. As results, the developed model was capable to provide better returns in fourteen of the twenty two periods under consideration. Besides, the average return considering all the periods was 0,03404 for the proposed model and -0,02086 for the IBOVESPA portfolio.
Keywords: Optimization. Investment. IBOVESPA

RESUMO
O mercado de ações tem crescido constantemente nos últimos anos, sendo que diversos índices também têm sido criados nesse mercado, como o Índice de Governança Corporativa (IGC), Índice de Sustentabilidade Empresarial (ISE) e IBOVESPA. Pensando no crescimento desse mercado, o presente trabalho tem por objetivo propor a construção de um portfólio

Recebido em 01.04.2014. Revisado por pares em 14.05.2014. Reformulado em 29.05.2014.
Recomendado para publicação em 31.08.2014. Publicado em 01.12.2014.

ótimo por meio da programação linear, tendo como base as empresas contidas simultaneamente nos índices IGC e ISE e compará-lo aos resultados obtidos por uma carteira real representada pelo IBOVESPA. As restrições do problema serão estabelecidas com base em indicadores do IBOVESPA. O modelo será criado de forma a atender as restrições estabelecidas e maximizar o retorno, comparando sempre com o retorno do IBOVESPA levando em consideração os anos de 2007 a 2012. Como resultados, o modelo desenvolvido foi capaz de fornecer melhores retornos em quatorze dos vinte e dois períodos analisados. Além disso, o retorno médio considerando todos os períodos em análise foi de 0,03404 para o modelo proposto e -0,02086 para a carteira do IBOVESPA.
Palavras-chave: Otimização. Investimento. IBOVESPA

RESUMEN
El mercado de valores ha crecido de manera constante en los últimos años, y muchos índices también se han creado en este mercado, como el Índice de Gobierno Corporativo (IGC), el Índice de Sustentabilidad Empresarial (ISE) y BOVESPA Index. Pensando en el crecimiento de este mercado, este trabajo tiene como objetivo proponer la construcción de una cartera óptima mediante programación lineal, basado en las empresas que figuran simultáneamente en los índices de la CIG y de ISE . Las restricciones del problema son establecidos con base en los indicadores del índice BOVESPA. El modelo se creó para cumplir con las restricciones y maximizar el retorno, siempre comparando con el regreso del BOVESPA Index teniendo en cuenta los años 2007 a 2012. Como resultado, el modelo fue capaz de entregar un mejor rendimiento en catorce de los veintidós períodos analizados. Por otra parte, la rentabilidad media de considerar todos los periodos de revisión y fue 0,03404 -0,02086 para la propuesta para el modelo IBOVESPA .
Palabras clave: optimización, IBOVESPA inversión

1. INTRODUCTION

The stock market has increasingly become a potential form of investment, not only for large and medium investors, but also for the small ones, as confirmed by Krukoski (2010, page 10): "Investments in shares and options in stock exchanges are becoming increasingly accessible to small investors, who seek an alternative source of income to financial independence."

However, financial investment decisions are taken in environments of great uncertainty. Within this context, two important variables must be taken into consideration when investing: risk and return of such investments (ASSAF NETO, 2010). Nonetheless, many investors have doubts about what the best assets to be acquired are, and, thus, how to build an investment portfolio that provides the highest return considering the lowest possible risk level (OLIVEIRA *et al.*, 2011). For this and other reasons, many mathematical models have been created to optimize investments in the stock market.

In addition to these models, some market indexes were also created in order to assess the performance of companies in different areas, serving as a basis of comparison between the observed returns. Indirectly, these indexes seek support in making investments in shares of those companies. Among the main existing indicators, which will also be the object of study in this work, it is possible to highlight CGI (Corporate Governance Index), CSI (Corporate Sustainability Index) and Bovespa Index (BOVESPA's Index).

Based on that, the problem which is the origin of the question that this article aims to answer is raised: is it possible to construct an optimized portfolio of stocks, with companies

belonging simultaneously to CGI and CSI so that this portfolio shows better results in terms of return and risk than the Bovespa Index's?

The hypothesis is that companies which attempt to simultaneously implement the best practices of corporate governance and corporate sustainability, by their characteristics, would present a superior performance to companies in the traditional market in terms of higher return and lower risk level. Thus, it is expected that transactions developed with these features present a higher index than those presented by the Bovespa Index, as well as a lower level of inherent volatility.

Considering this context, the objective of this paper is to propose the construction of an optimal portfolio by means of linear programming methods, based on the companies contained simultaneously in the CGI and CSI indices and compare it to the results obtained by a real portfolio represented by the Bovespa Index.

For this purpose, the technique used is the optimization based on a mathematical model of linear programming. The companies in the elaborated portfolio will be the same contained in the CGI and CSI indicators, simultaneously. The restriction criteria established are related to the liquidity of the company, contribution margin, volatility, and the sectors in which it operates, and it relies on the comparison of values obtained through the Bovespa Index. Once the information that will serve as constraints is collected, the linear programming model will decide which companies to invest in and how much to invest in each of them so that they maximize the return.

2. THEORETICAL REFERENCES

2.1 Investment in the Stock Market

Junior Motta *et al.* (2007) argue that among the many complex decisions in our daily lives, one of them is related to the investment of resources; in other words, application of limited resources in order to provide an adequate return to investors. A basic premise in this context is that capital owners always want the highest return and the lowest risk for any type of investment to be made. Referring to investments in the stock market, specifically regarding the selection of portfolios, a complex decision is based on the choice of which assets to invest and in what proportion. According to Gitman and Madura (2003), the portfolio selection is an analysis of how you can invest resources in more than one action (diversification) in order to obtain the greatest financial return associated with the lowest risk.

The pioneering work in the area of portfolio optimization actions was the proposition of the mean-variance model from Markowitz. Known as the "Portfolio Theory", the work of Markowitz (1952) inaugurated what was called Modern Finance (HAUGEN, 1997). This proposition assumes that the investor, the expected return, and the volatility of possible returns are crucial in defining the optimal portfolio (SOARES, 2011). As stated by Bartholomew-Biggs (2005), the problem is formulated either to minimize the risk of the portfolio for a given level of return required by investors or to maximize the level of expected return of the portfolio associated with a given level of risk. So, such concepts have emerged from the need for a practical rule for rational investors to allocate their resources in investment portfolios (CASTRO JR; FAME, 2002).

However, the diversification of the portfolio proposed by Markowitz, despite the advancements made in understanding the risk-return binomial, affects the process of decision making to an even more complex and often conflicting action, because, in these terms, the construction of an optimal investment portfolio shall involve an even more full-of-alternatives scenario to be studied. Very often, these alternatives relate different points of view and ways

of considering choices. This process takes into account the multiplicity of factors directly or indirectly related to the decision process (SOARES, 2007).

As a mean for assisting the decisions of investors, in the sense of creating benchmarks and monitoring performance tools of a set of shares, São Paulo Stock Exchange (BM & FBOVESPA) created several indices over the years, among them: the Bovespa Index (IBOVESPA), CGI and CSI.

2.1.1 São Paulo Stock Exchange Index - Bovespa Index

The Bovespa Index (IBOVESPA) is the average indicator of the market quotations of the shares of the Stock Exchange from São Paulo. This index was created in 1968 and, since then, has a representation to demonstrate the performance of the assets most traded on the BM & FBOVESPA (BM & FBOVESPA, 2014a).

The notoriety of the index can be analyzed in terms of liquidity and market capitalization, considering that in order to integrate the theoretical portfolio companies have to meet, cumulatively, the following criteria (BMF & BOVESPA, 2014th, p.2):

a) It is necessary to be among the eligible assets that during the term of the three (3) previous portfolios, in descending order of the Negotiability Index (IN), collectively account for 85% (eighty five percent) of the total sum of those indicators;
b) To attend the trading floor 95% (ninety five percent) of the times in the period of the three (3) previous portfolios;
c) To have greater participation or equal to 0.1% (zero point one percent) in terms of financial volume in the cash market (round lot) in the period of the three (3) previous portfolios.

It is noteworthy that, every four months in the periods from January to April, May to August and September to December, the theoretical portfolio of the Bovespa Index is reviewed and a share may be excluded if the composition does not meet the criteria adopted by the methodology of calculation from BM & FBOVESPA.

2.1.2 – Corporate Governance Index (CGI)

The first decade of the 2000s marked the country because of deeper discussions about minority shareholders and good corporate governance. At the end of 2000, such idea had its main incentive with the creation of CGI by BM & FBOVESPA, which has been calculated since 2001. BM & FBOVESPA (2014b) defines the CGI as "Stock Index Corporate Governance that aims to measure the performance of a theoretical portfolio composed of shares of companies with good corporate governance levels." CGI includes companies that have good corporate governance and is classified into three levels: level 1, level 2 and new market (KERR, 2011).

Companies listed on Level 1 of Corporate Governance must provide improvements in sharing information with the market and they promote dispersion of shareholder control (ALMEIDA; SCALZER; COSTA, 2006). Companies listed on level 2 commit to fulfill the requirements for level 1 and, additionally, adopt a set of broader rules for governance practices, prioritizing and expanding the rights of the minority shareholders. Besides that, the companies of the new market follow some specific rules, even broader than those of the second level of governance and additional to those required by the Brazilian law rules, as shown in the text below:

> To join the New Market, the company must sign a contract in which it is committed to use a set of corporate rules more stringent than those found in the Brazilian legislation. This set of rules is termed as good corporate governance practices. Such rules increase shareholders' rights and enhance the quality of information provided to external users (ALMEIDA; SCALZER; COSTA, 2006).

Rogers, Ribeiro, and Souza (2005, p 55.) state that:

> [...] the insertion of higher standards of governance increases the return, reduces the return volatility, increases trading volume and liquidity, and reduces the exposure of stock returns to external risks, which consequently makes the costs of capital lower and raises the value of the company [...].

Another important aspect of good governance is to provide a better and more direct relationship between managers and owners, as stated by Vieira and Mendes (2006, p 104):

> [...] the practice of good governance in institutions appears as a mechanism to provide greater transparency to all stakeholders in the enterprise, minimize existing information asymmetry among managers and owners, and make the shareholders who do not belong to control block reduce their losses in case of a possible sale of the company [...]

Several works have been trying to deepen their knowledge about CGI and to associate the implementation of best corporate governance practices to better performance in the financial market. Among these studies, it is possible to highlight the work of Smith and Kalatzis (2006) who concluded that firms that implement differentiated corporate governance levels, in other words, companies that make up the portfolio of CGI, have higher average returns than the average returns of the Bovespa Index portfolio. In a recent national work, Pinheiro (2013) characterized the historical series as correlated, uncorrelated, and anti-correlated for companies from CGI, using the *Detrended Fluctuation Analysis* method. The study addressed the historical series of shares from the Bovespa Index and CGI from BMF & BOVESPA from January 2, 2007 to December 31, 2012. As a result, it was possible to identify that the group that is formed by stocks from companies that have the characteristic of corporate business practices is often preferred by investment managers who seek abnormal gains for having the largest percentage in long-range correlated shares. It is also possible to mention, in this context, the work of Antonelli *et al.* (2014), which sought to determine the window of event that best explains the relationship between adhesion and migration in relation to the Differentiated Levels of Corporate Governance from BM & FBOVESPA and the valuation of shares in companies. Also, the work of Beiruth *et al.* (2014) investigated the association between the level of corporate governance of companies, measured by different segments of BM & FBOVESPA, and the opportunity in the disclosure of financial reports to the market.

2.1.3 Corporate Sustainability Index (CSI)

CSI was established in 2005 aiming to "create an investment environment compatible with the demands of sustainable development of contemporary society and stimulate the ethical responsibility of corporations." (BM&FBOVESPA, 2012). Also according to the BM & FBOVESPA (2012), "CSI is a tool for benchmarking the performance of companies listed in the BM & FBOVESPA under the aspect of corporate sustainability based on economic efficiency, environmental balance, social justice, and corporate governance" (BM & FBOVESPA, 2012). To Marcondes and Bacarji (2010, p.19), "CSI was the fourth stock index in the world created with the objective of showing the market performance of a portfolio of businesses that adopt the principles of sustainable management."

The Social Sustainability Index is composed of a stocks portfolio whose companies are subject to certain restrictions. CSI "consists only of shares of companies committed to sustainability. When accepted at this index, the company is actually receiving a certificate of sustainability" (KERR, 2011, p.97).

In this scenario, in order to make the decision-making process even more rational and cohesive, several mathematical methods were created with regard to assisting in choice-making, assuring that the chosen option would be the most coherent alternative (GUEDES *et al.*, 2011). Within this context, the techniques of operational research are included.

2.2 Operational research applied to finances

Barbosa and Zanardini (2010, p.28) define operational research as:

> A practical science, establishing reliable decision parameters, which considers the factors and scenarios of problems and establishes, through mathematical models, the ability to visualize possible solutions to situations whose variables, constraints and objective function are read from calculations; being these calculations modeled from phases of problem structuring.

One of the methods used in solving optimization problems is the linear programming, the method used in this work. According to Lachtermacher (2009, p.20) "a linear programming problem is a mathematical programming problem in which the objective function and constraints are linear." For Barbosa and Zanardini (2010, p.28) the linear programming "consists in modeling and solving optimization problems of a linear function in face of constraints which are also linear."

With the rapid development in the processing capacity of computers, operations research started to be used in various sectors, such as construction, economy, production, finance and others. According to the area of operation, different mathematical models were formulated for solving certain problems, all with the same logic optimization of results, whether profit maximization or cost minimization, for instance.

Linear optimization models for determining an optimal portfolio can consider various constraints inherent in the process and still provide simple solutions in addition to being computationally fast (PAPAHRISTODOULOU, 2004). Several papers have been published with the development of models (RENDLEMAN, 1995; SOARES, 2011; FABOZZI et al, 2007; REILLY, 2002) seeking to solve the problem of building an optimal portfolio. Some of these papers (CESARONE et al, 2010) sought to introduce some alternative risk measures such as the mean-absolute deviation model of Konno and Yamazaki (1991), applied to situations in which returns have asymmetric distributions (ZENIOS; KANG, 1993). Variations may be found in the work of Loraschi *et al.* (1995) for the unconstrained

optimization problem, Crama and Schyns (1999) with restrictions of upper and lower limits and Perold (1984) and Bienstock (1996) for formulations with non-linear objective function. In addition to changes in modeling, different solving approaches were used over the years, from classic methods such as branch-and-bound for integer problems (Bienstock, 1996) to heuristic procedures based on genetic algorithms (LORASCHI *et al.*, 1995) and *simulated annealing* (CRAMA; SCHYNS, 1999). In recent years, Das *et al.* (2010) integrated the mean-variance portfolio theory of Markowitz with the behavioral theory of Shefrin and Statman to propose a structure of portfolios based on a new definition of risk. Lai *et al.* (2011) presented a new approach for estimating the mean and covariance for the portfolio optimization from the mean-variance model of Markowitz.

In the national context, the work of Oliveira *et al.* (2011) addressed the optimization of the risk/return relation of a portfolio of assets that participated in the Bovespa Index in its last eight amendments. As a result, the optimized portfolio with 11 assets obtained a return of 0.17% with a risk of 1.14, indicating better results than those presented by other indices. Following the same line of reasoning of this work, Sirqueira and Kalatzis (2006) used linear programming to check whether a portfolio with shares of companies that adopt corporate governance practices provides greater return to the investor than a portfolio of stocks of companies that do not adopt this set of practices. The authors conclude the work stating that there is a difference between the returns provided by each of these portfolios, which may indicate that companies that implement such practices provide a higher return. In the same context, we can mention the works of Ignatius *et al.* (2012) with the application of a linear programming model based on the work of Papahristodoulo (2004) and the Black-Scholes model to determine the best stock portfolio, and Lima *et al.* (2008), who studied 61 assets in the composition of the Bovespa Index during the first quarter of 2008 using graph theory. This paper presents a similar approach, working with companies belonging to CSI and CGI indices, detailed next.

3. RESEARCH METHODOLOGY

According to the classification proposed by Gil (2008), this research is characterized by the following aspects: regarding its nature as applied research; regarding its approach as quantitative research, and its objectives as explanatory research.

3.1 - Definition of the sample

For the definition of the companies that formed the sample of this study, at first, we considered all companies traded on BM & FBOVESPA from 2007 to 2012, leading to a total of 603 companies. Thereupon, those companies which were still active and had stocks traded during the reference period were selected, which, in turn, reduced the initial sample to 365 companies.

So, for the 365 selected companies, quarterly data (from 2007/01 to 2012/03) regarding a liquidity indicator (general liquidity), profitability (net margin), and risk (volatility) was collected, as well as the return of actions that served as constraints in the optimization process. All data was collected in the Economática software. In this process, the companies that had missing data, i.e, those which did not disclose all their data regarding the collected indicators, were also excluded.

Finally, from the final amount of companies to which certain filters have been applied, the companies that were present in both CGI and CSI were selected. After the initial collection of the available data, taking into account the constraints presented, a universe of 22,

20, 24, 26, 29 and 30 companies was used from 2007 to 2012, respectively. On Appendix I it is possible to identify the companies that were selected for investment by the optimization model in each of the periods of analysis.

.

3.2 - Procedures for modeling and optimization

With the companies' portfolio defined, and the indicators of liquidity, profitability, risk and return collected, the optimization model was implemented. Such process sought to build an optimal portfolio of shares, for each quarter, taking into account the restrictions based on the indicators collected.

The supplement Solver from Excel was used to calculate the "optimal investment portfolio". It is important to mention that the analysis was done comparing the returns in $p + 1$ from the model resolution in period p, i.e., the model uses data from period p for optimization and compares the results with the Bovespa Index portfolio for the period $p + 1$. For each quarter we have:

a) Sets

I: $set\ of\ companies\ i$
S: $set\ of\ sectors\ j$

b) Data

r_i: $company\ i\ return$
l_i: $company\ i\ general\ liquidity$
m_i: $company\ i\ net\ margin$
v_i: $company\ i\ volatility$
l^*: $Bovespa\ Index\ portifolio\ general\ liquidity$
m^*: $Bovespa\ Index\ portfolio\ net\ margin$
v^*: $Bovespa\ Index\ portfolio\ volatility$
s_j^*: $percentage\ invested\ in\ sector\ j\ by\ Bovespa\ Index\ portfolio$

c) Decision Variable

x_i: $percentage\ invested\ in\ company\ i$

d) Objective function

$$Maximize\ Return\ Portfolio = \sum_i x_i \times r_i \qquad (1)$$

e) Constraints

$$\sum_i x_i \times l_i \geq l^* \qquad (2)$$

$$\sum_i x_i \times m_i \geq m^* \qquad (3)$$

$$\sum_i x_i \times v_i \leq v^* \quad (4)$$

$$\sum_{i \in S_j} x_i \geq s_j^* \quad \forall j \quad (5)$$

$$\sum_i x_i \leq 1 \quad (6)$$

$$x_i \geq 0 \quad \forall i \quad (7)$$

The objective function (1) seeks to maximize the portfolio return, constraint (2) ensures that the portfolio general liquidity is at least equal to the Bovespa Index portfolio liquidity, constraint (3) ensures that the portfolio net margin is at least equal to the Bovespa Index portfolio margin, constraint (4) ensures that the portfolio volatility is at most equal to the Bovespa Index portfolio volatility, constraint (5) ensures that the total amount invested in each sector is at least equal to the total invested in the same sector in relation to the Bovespa Index portfolio, constraint (6) ensures that the investment will not be greater than 100% and constraint (7) ensures that the percentage investment in each company is greater or equal to zero.

4. RESULTS PRESENTATION AND ANALYSIS

Table 1 presents the results for the optimized portfolio returns in each quarter from 2007 to 2012. The first column shows the period in question. The second, the return of the portfolio according to the Bovespa Index, and the third, the portfolio return from the developed mathematical model. References to specific investments in a particular company can be checked on tables I.1 and I.2 at the end of the work, in the Appendix I.

Table 1 - Comparative returns of the Bovespa Index portfolio with the optimized portfolio (1st quarter/ 2007 to 3rd quarter/ 2012)

Period	Bovespa Index Return	Optimized Portfolio Return	Period	Bovespa Index Return	Optimized Portfolio Return
2007 – 1st Q	-	-	2010 – 1st Q	-0.01768	-0.07286
2007 – 2nd Q	0.25595	0.32147	2010 – 2nd Q	-0.25945	0.01219
2007 – 3rd Q	0.09255	0.07030	2010 – 3rd Q	0.17158	0.22128
2007 – 4th Q	-0.00851	0.16073	2010 – 4th Q	-0.06292	-0.09959
2008 – 1st Q	-0.10249	0.04855	2011 – 1st Q	-0.08052	0.01400
2008 – 2nd Q	0.00919	-0.11850	2011 – 2nd Q	-0.18454	-0.15436
2008 – 3rd Q	-0.36453	-0.45278	2011 – 3rd Q	-0.31170	-0.22553
2008 – 4th Q	-0.47406	-0.19826	2011 – 4th Q	0.08577	0.11991
2009 – 1st Q	0.14869	0.04450	2012 – 1st Q	0.17262	0.14195
2009 – 2nd Q	0.29221	0.20236	2012 – 2nd Q	-0.33396	0.00438
2009 – 3rd Q	0.23220	0.37444	2012 – 3rd Q	0.11368	0.03502
2009 – 4th Q	0.16690	0.29964			

Source: own elaboration

In fourteen out of the twenty-two periods (the 1st quarter of 2007 was the basis for the calculation of the portfolios for the following quarters) the return of the theoretical portfolio outperformed the return of the Bovespa Index portfolio, i.e., the model used proved to be quite efficient regarding the optimization of the hypothetical portfolio developed. The average of the periods also confirms this statement. The average of returns of the theoretical portfolio was 0.03404 for the period, while the Bovespa Index was virtually zero, closing at -0.02086. The sum of the returns for all periods under review was 0.74884 for the portfolio and -0.45902 for the Bovespa Index.

By analyzing the detailed description for the second quarter of 2007, it can be seen that the return of the optimized portfolio is greater than the Bovespa Index return. The largest share is in BANK OF BRAZIL and PETROBRAS (Table I.1 from Appendix I), since such companies have a relatively high return (focusing on the high level of liquidity). In the third quarter of 2007, the return of the optimal portfolio was lower, reaching 0.07030. NATURA and SABESP negative results, besides the beginning of the crisis, caused the return of the portfolio to be reduced at about 22%. For the fourth quarter of 2007, even though the market presents a sharp drop, as shown by the Bovespa Index itself, the portfolio achieved a good result in this period, with an average return of 16%. This return is well above the average of the Bovespa Index, which closed negative at about -1%. Companies such as PETROBRAS, ELETROPAULO and WEG contributed considerably to this good result.

For the first quarter of 2008, despite presenting a reduction in the return, the optimized portfolio still outperformed the Bovespa Index. It is noteworthy that in this period it was not invested 100% of the capital, since some companies contained in the 2007 portfolio were not in the 2008 one, as seen in Tables I.1 and I.2 from Appendix I. The crisis made all portfolio companies have negative returns, hindering the portfolio optimization that "did not maximize profit, but minimized the loss". In the second quarter of 2008, the hypothetical portfolio did not produce good results, since the companies that outperformed the previous period and those which had the highest participation in the portfolio in the second quarter of 2008 had a large deficit in their return. NATURA, whose return was 0.1 in the first period of 2008, and plunged to -0.3, a reduction of almost 33%, largely stands out. For the third quarter of 2008, most companies still present negative returns, which caused a poor result in the average return of the portfolio. Another relevant factor is the increase of the companies' volatility, highlighting mainly Gerdau, which had a return of 0.5 in the previous period and dropped to -0.9. For this company, the volatility changed from 44.3 to 70.3. These figures show the impact of the crisis on corporate outcomes. In the last quarter of 2008, the effects of the crisis are still evident. It can be observed that the same companies which returned to profit in the previous period were at a loss. All portfolio companies had a negative return, including the BOVESPA Index average itself, showing a fall in the stock market as a whole. Although the constructed portfolio had presented a negative return, it stood out the Bovespa Index on account of having less significant losses.

In regard to the first quarter of 2009, companies began to show better results, originating a portfolio with a positive return. The estimated model in the last quarter of 2008 indicated an investment of 23% in SABESP and about 10% in ITAU UNIBANCO. These companies have not achieved good results in the current period of analysis; mainly SABESP, which had a negative return of -0.1. In the second quarter of 2009 a great improvement in the portfolio return was noticed, in spite of not having overcome the Bovespa Index, its return has greatly improved in comparison to the previous period. The companies have started to recover from the crisis. The ones which stood out in this period were BRASKEM and USIMINAS. Both companies had presented negative returns in the prior period and in the current period they had the highest return among the companies under study. This was one of the reasons

why the portfolio has not overcome the Bovespa index, since the model is based on the previous optimization period, and as these companies did not have good results, the model indicated that there should not be investments in those companies. In the third quarter of 2009 the return of the portfolio increased significantly. The main investment was in BRADESCO, about 18% of the capital, which had a return of 0.3. The companies are still recovering from the crisis, their volatility indices are still extremely high. In the last quarter of 2009, the return, despite the fact that it decreased in comparison to the last period, still had a good result, whereas the Bovespa index experienced a slight decrease (from 0.23 to 0.17). The portfolio margin increased significantly as to the last period along with the liquidity, which improved about 47%.

In the first quarter of 2010, both the hypothetical portfolio and the Bovespa Index returns fell considerably. One of the reasons is that a large amount of companies turned to negative returns again, with a small increase in volatility and a big drop in liquidity ratios and contribution margin. The crisis still has a negative effect on the companies. With regard to the second quarter of 2010, despite having obtained an extremely low return during this period, the portfolio still beat the Bovespa index, which had a negative return of -0.26. A very significant indicator in this period was the liquidity of companies, which increased about 21%. The volatility declined in that period and the portfolio had a brief improvement. In the third quarter of 2010, the portfolio had a great improvement in its return, going from 0.01 to 0.22. Some companies that stood out were the EVEN and SULAMERICA, which had a significant increase in their return. EVEN was driven by the solid growth in the construction sector. In the last quarter of 2010, the period of "post-crisis", companies began to show some instability, often associated with high levels of volatility.

In the first quarter of 2011, the portfolio return was again positive, in contrast to an extremely low value, only 0.01. Other indicators showed no significant change. The company CPFL ENERGIA presented a very good return in this period even though the model did not invest in this company. In the next quarter, the returns fell again, both the portfolio's and the Bovespa Index's. In spite of providing a good liquidity indicator in average, the portfolio is still not efficient in this period as it has a negative return. In the third quarter of 2011, the return continues to fall, highlighted by a sharp decline in the Bovespa Index. It can be observed that not only the portfolio companies are having a bad result, but the market itself is falling. The liquidity of the companies fell as well as the volatility, going from 27.25 to 24.56. In the last quarter of 2011, the portfolio, and even the Bovespa Index, showed a significant improvement in their returns; the former going from -0.23 to 0.12. Its liquidity remained practically constant, and the margin showed a slight decrease from one quarter to another.

Finally, despite the fact that the return increased over the past period, in the first quarter of 2012, the portfolio had a lower return than the Bovespa Index's. On the other hand, the optimized portfolio showed excellent margin, liquidity and volatility indicators compared to the Bovespa Index. In the second quarter of 2012, the return fell again, notwithstanding still above the return submitted by the Bovespa Index portfolio. In the third quarter of 2012, the return shows a slight increase, though not proportional to the increase of the Bovespa Index portfolio. The liquidity, volatility and margin in this period showed excellent results in relation to the Bovespa Index.

In annual terms it is possible to verify that in all the years the portfolio proposal was superior to the Bovespa Index portfolio, as shown in Table 2, while in terms of volatility, the portfolio has always been lower than the Bovespa Index's, as seen in Table 3.

Table 2 - Comparison of annual returns of the Bovespa Index portfolio with optimized portfolio

Period	Bovespa Index Return	Optimized Portfolio Return	Difference
2007	0.3400	0.5525	0.2125
2008	-0.9319	-0.7210	0.2109
2009	0.8400	0.9209	0.0809
2010	-0.1685	0.0610	0.2295
2011	-0.4910	-0.2460	0.2450
2012	-0.0477	0.1814	0.2290

Source: own elaboration

Table 3 - Comparison of the annual volatility of the Bovespa Index portfolio with the optimized portfolio

Period	Bovespa Portfolio Volatility	Optimized Portfolio Volatility	Difference
2007	32.0429	30.8825	-1.1604
2008	50.5765	46.5767	-3.9998
2009	40.1349	35.0387	-5.0961
2010	28.6471	27.5587	-1.0884
2011	32.8107	29.8422	-2.9684
2012	35.9077	31.3197	-4.5881

Source: own elaboration

Figure 1 presents the portfolio returns and the Bovespa Index returns in the analyzed period:

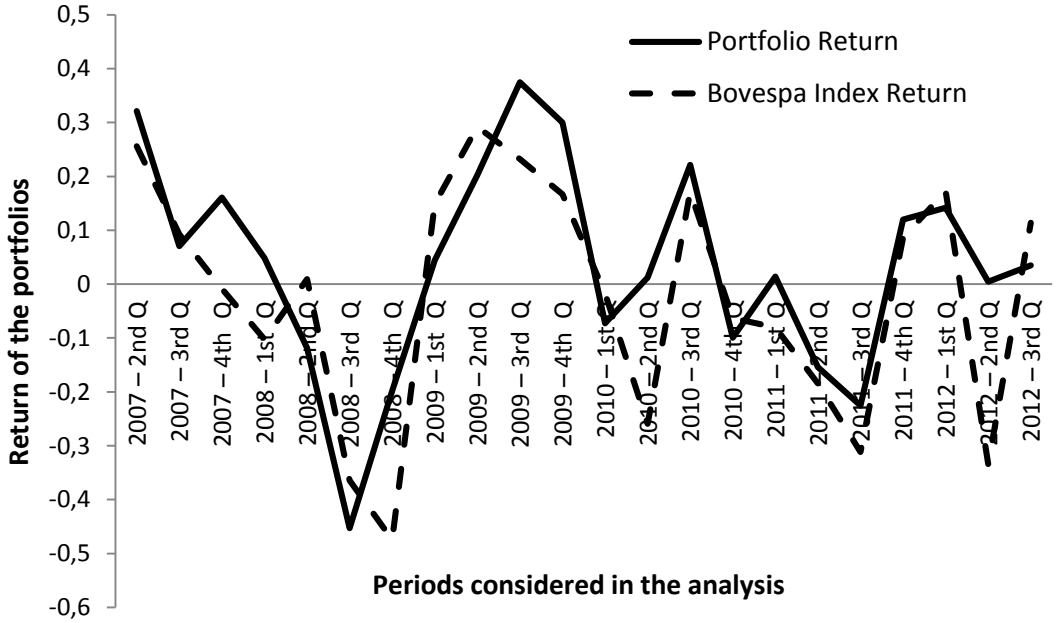

Figure 1 – Graph from the Returns of the Optimized Portfolio / Bovespa Index Returns

By analyzing the graph, it is possible to notice that the two series follow similar trends of growth and decline. It is observed that the return of the portfolio, in most analyzed periods,

is above the return of the Bovespa Index. From the fourth quarter of 2011 on, the portfolio return is more uniform, with less fluctuation. Another interesting point is that the negative returns virtually reduce the positive returns to zero.

Table 4 - Number of participations in the optimized portfolio

Companies	Number of participations in the optimized portfolio	Companies	Number of participations in the optimized portfolio
NATURA	23	ODONTOPREV	14
LIGHT Inc.	22	ENERGIAS BR	13
SUZANO PAPEL	20	GERDAU	13
BRASKEM	19	IOCHP-MAXION	13
ECORODOVIAS	19	WEG	13
TIM PARTC. Inc.	19	CPFL ENERGIA	12
VALE	19	SULAMERICA	12
EMBRAER	18	USIMINAS	12
ITAUBANCO	16	ITAUSA	11
TRACTEBEL	16	GERDAU MET	8
EVEN	15	COPEL	7
PETROBRÁS	15	BRADESCO	6
BRASIL	14	DURATEX	6
BRF FOODS	14	SABESP	6
CCR RODOVIAS	14	CEMIG	4
COPASA	14	ELETROBRÁS	4
FIBRIA	14	LIGHT Inc.	3

Source: own elaboration

Furthermore, Table 4 presents companies that were used in the development of the hypothetical portfolio and how many times, throughout the simulations, such companies actually composed this portfolio.

5. CONCLUSION

This work aimed to propose the construction of an optimal portfolio through linear programming, based on the companies contained, simultaneously, in the CGI and CSI indices and compare it to the results obtained by a real portfolio represented by the Bovespa Index.

In this sense, the results showed that, in fourteen out of the twenty-two periods, the return of the theoretical portfolio outperformed the Bovespa Index return, in other words, the model used was very efficient in optimizing the hypothetical portfolio. The average of the periods also confirms this statement. The average return of the theoretical portfolio was 0.03404 for the period, while the Bovespa Index was virtually zero, closing at -0.02086. The sum of the returns for all periods under review was 0.74884 for the portfolio, and -0.45902 for the Bovespa Index.

The fact that the average returns have given such low values can be explained by the crisis present in most of the analyzed periods. Considering that the Bovespa Index is the largest benchmark among all indices from BM & FBOVESPA, and the portfolio meets at least the same restrictions of this index, it is possible to state that it makes the portfolio even more reliable. The companies presented in the portfolio are the most liquid ones, whose net margins are higher and the volatility is lower.

Those companies, according to the theory, have better performance in the market. In 2007, until mid-2008 (when the crisis started), the portfolio showed much better results than the Bovespa Index's. However, the crisis had a greater impact on the economy in August and September 2008, with the stagnation of the giants of the personal loan. Making an analysis of this period, the hypothetical portfolio did not present very satisfactory results, with a return below the Bovespa Index, since the second period of 2008 until the second period of 2009. This means that the model did not react well to this period of crisis, which may be explained by the fact that companies tend to present a high level of volatility. Since the portfolio is optimized in *p* for investments in *p+1*, the results of companies in *p* widely varied to *p + 1*, increasing the difficulties for optimization and, consequently, the results of the theoretical portfolio.

The objectives proposed in the work have been achieved since the model maximized the return on the portfolio, given the established constraints, and exceeded the average return of the Bovespa Index portfolio in the analyzed periods.

Nonetheless, it is important to point out that some limitations were observed throughout the research. The first one is regarding the available data. Even though the Economática Software has been used, compiling and presenting a great infinity of data related to the companies, many of these companies are not interested in sharing relevant information and financial indicators with the public. Thus, the sample is quite low, since many companies do not have enough data as required for the study. Furthermore, this work opted to use only one method of optimization, seeking evidence of whether the characteristics of firms that invest in Corporate Governance and Sustainability are determining factors to a better return and lower risk level. However, other methods could also be used for this purpose.

Although the presented results confirm the hypothesis that, in general, firms which engage in adopting simultaneously the best practices of corporate governance and corporate sustainability show better indicators, both in terms of return and risk, new studies can deepen this type of analysis, expanding the horizon to be studied, as well as using other types of indicators and restrictions. Moreover, there is also the possibility of using different types of optimization models, such as Kruskal's, Prim's, and Dijkstra's algorithm, among others, in order to compare what the best optimization model would be as to fit the nature of the referred data.

ACKNOWLEDGMENTS

The authors would like to thank the anonymous reviewers for their valuable comments and suggestions to improve the quality of the paper. They are also grateful to Bruna Cunha and Milena Marques for their support in the translation of the article.

REFERENCES

ALMEIDA, J. C. G.; SCALZER, R. S.; COSTA, F. M. **Níveis diferenciados de governança corporativa da BOVESPA e grau de conservadorismo: estudo empírico em companhias abertas listadas na BOVESPA. Revista de Contabilidade e Organizações, v. 2, n. 2, p. 117-130, 2008.**

ACIONISTA.COM.BR. Governança Corporativa - Cotações. Disponível em: http://www.acionista.com.br/governanca/cotacoes.htm. Acesso em 23 fev. 2013.

ANTONELLI, R. A.; JESUS, M. C.; CLEMENTE, A.; CHEROBIM, A. P. M. S.; SCHERER, L. M. A adesão ou migração aos níveis diferenciados de governança corporativa e a valorização acionária. **Revista de Gestão, Finanças e Contabilidade**, v. 4, n. 1, 2014, p. 52-76.

ASSAF NETO, A. **Mercado Financeiro**. São Paulo: Atlas, 9ª ed., 2010. 339 p.

BARBOSA, M. A.; D. ZANARDINI, R. A. **Iniciação à pesquisa operacional no ambiente de gestão.** Curitiba: **Ibpex**, 2010. 160 p.

BARTHOLOMEW-BIGGS, M. Nonlinear optimization with financial applications. Springer, 2005. 261 p.

BEIRUTH, A. X.; BRUGNI, T. V.; FÁVERO, L. P.; GOES, A. O. S. Níveis diferenciados de governança corporativa e *disclosure timeliness*: um estudo exploratório no mercado brasileiro. **Revista de Gestão, Finanças e Contabilidade**, v. 4, n. 1, 2014, p. 77-89.

BIENSTOCK, D. Computational study of a family of mixed-integer quadratic programming problems. **Mathematical Programming,** v.74, p.121-140, 1996.

BM&FBOVESPA. **Índice de Sustentabilidade Empresarial - ISE.** Disponível em: <http://www.bmfBOVESPA.com.br/indices/ResumoIndice.aspx?Indice=ISE&Idioma=pt-br>. Acesso em: 23 fev. 2013.

BM&FBOVESPA. **Metodologia do Índice BOVESPA**. São Paulo: BM&FBOVESPA, 2014a.

BM&FBOVESPA. **Índice de Governança Corporativa. 2014b.** Disponível em: <http://www.bmfbovespa.com.br/Indices/ResumoIndice.aspx?Indice=IGC&Idioma=pt-br>. Acesso em: 23 fev. 2014.

CASTRO Jr., F.; FAMÁ, R. As novas finanças e a teoria comportamental no contexto da tomada de decisão sobre investimentos. **Caderno de Pesquisas em Administração**, São Paulo, v. 9, n 2, 2002.

CESARONE, F.; SCOZZARI, A.; TARDELLA, F. Portfolio selection problems in practice: a comparison between linear and quadratic optimization models. **Quantitative Finance Papers**, 2010.

CRAMA, Y.; SCHYNS, M. Simulated annealing for complex portfolio selection problems. Working paper. Belgium: University of Liège, 1999.

DAS, S.; MARKOWITZ, H; SCHEID, J.; STATMAN, M. Portfolio optimization with mental accounts. **Journal of Financial and Quantitative Analysis**, v. 45, n. 2, 2010.

FABOZZI, K. F. J.; PACHAMANOVA, P. N.; FOCARDI, S. M. Robust portfolio optimization and management: Jonh Wiley & Sons, 2007.

GIL, Antônio Carlos. **Como elaborar projetos de pesquisa**. 4. ed. São Paulo: Atlas, 2008.

GITMAN, L. J; MADURA, J. **Administração financeira**: uma abordagem gerencial. São Paulo: Pearson, 2003.

GUEDES, P. H. D.; BRANDALISE, N.; SILVA, R. T.; CARNEIRO, C. A. G. V. Tomada de decisão de investimento em imóveis no estado do Rio de Janeiro. In: Simpósio Brasileiro de Engenharia de Produção, XVIII, 2011, Bauru/ SP.

HAUGEN, R. A., **Modern Investiment Theory**. California: Pretince Hall. 1997.

IGNÁCIO, A. A. V.; COELHO, S. T. MEZA, E. B. M. VIANNA, D. S. Modelo de otimização aplicado ao mercado de opções. In: Simpósio Brasileiro de Engenharia de Produção, XIX, 2012, Bauru/ SP.

KEER, R. B. **Mercado Financeiro e de Capitais.** São Paulo: Pearson Education do Brasil, 2011. 248 p.

KONNO, H; IAMAZAKI, H. Mean-absolute deviation portfolio optimization model and its applications to Tokyo Stock Market. Management Science, v. 37, p. 519-531, 1991.

KRUKOSKI, F. A. **Programação linear aplicada ao mercado de opções na criação de um portfólio seguro.** 2010. 87 f. Dissertação (Mestrado em Ciências) - Universidade Federal do Paraná, Curitiba, 2010.

LACHTERMACHER, G. **Pesquisa Operacional na Tomada de Decisões.** 4ª ed. São Paulo: Pearson Education do Brasil, 2009. 224 p.

LAI, T. L.; XING, H.; CHEN, Z. Mean-variance portfolio optimization when means and covariances are unknown. *The Annals of Applied Statistics*, v. 5, n. 2, 2011, p. 798-823.

LIMA, L. S.; ROSA, P. E. B.; MOURA, V. F. B. Teoria dos grafos aplicada ao mercado acionário brasileiro. In: Simpósio Brasileiro de Engenharia de Produção, XV, 2008, Bauru/ SP.

LORASCHI, A.; TETTAMANZI, A.; TOMASSINI, M.; VERDA, P. Distributed genetic algorithms with an application to portfolio selection problems. In Artificial neural nets and genetic algorithms, Pearson, D.W., Steele N.C., and Albrecht, R.F. (eds), (1995) 384-387.

MARCONDES, A. W.; BACARJI, C. D. **ISE:** sustentabilidade no mercado de capitais. São Paulo: Report Editora, 2010. 173 p.

MARKOVITZ, H. M. Portfolio selection. **Journal of Finance**, v. 7, 1952, p. 77-91.

MINARDI, A. M. A. F. Retornos passados preveem retorno futuros? **RAE-eletrônica**, v. 3, n. 2, p. 2-18, 14 jul. 2004. Disponível em: <http;//www.rae.com.br/el14etronica/index.cfm?FuseAction=Artigo&ID=1738&Secao=FINANCAS&Volume=3&Numero=2&Ano=2004>. Acesso em: 08 dez. 2012.

MOTTA JÚNIOR, N.; OLIVEIRA, U. R.; GUTIERREZ, R. H. Minimização de riscos de investimentos em carteiras de ações através da pesquisa operacional. In: IV SIMPÓSIO DE EXCELÊNCIA EM GESTÃO E TECNOLOGIA, 4, 2007, Rio de Janeiro.

OLIVEIRA, M. R. G.; CARVALHO, K. S.; ROMA, C. M. S. Otimizando uma carteira de investimentos: um estudo com ativos do IBOVESPA no período de 2009 a 2011. In: Simpósio Brasileiro de Engenharia de Produção, XVIII, 2011, Bauru/ SP.

PAPAHRISTODOULOU, C. Options strategies with linear programming. **European Journal of Operational Research**, v. 170, p. 246-256, 2004.

PEROLD, A.F. Large-scale portfolio optimization. **Management Science**, v.30, p.1143-1160, 1984.

PINHEIRO, C. A. O. Ganhos anormais através das séries históricas de ações de empresas com práticas empresariais corporativas no Brasil. **Revista de Gestão, Finanças e Contabilidade**, v. 3, n. 2, 2013, p. 121-133.

REILLY, F. K.; BROWN, K. C. Investiment analysis portfolio management; South-Western College Pub, 2002.

RENDLEMAN, R. J. An LP approach to option portfolio selection. Advances in Futures and Options Research, v. 8, p. 31-52, 1995.

ROGERS, P.; RIBEIRO, K.; SOUSA, A. F. Comparações múltiplas das carteiras de bolsa no Brasil: avaliação da performance do índice de governança corporativa. **Revista de Gestão USP** (REGE-USP), São Paulo, v.12, n.4, p.55-72, 2005.

SIQUEIRA, A. B.; KALATZIS, A. E. G. Otimização de portfólio de ações: uma análise comparativa entre o IGC e o IBOVESPA. In: Simpósio Brasileiro de Pesquisa Operacional, 38, 2006, São Paulo.

SOARES, A. Tomada de decisão – breve reflexão. Artigo publicado em 18 de dezembro de 2007. Disponível em: <http://pt.shvoong.com/exact-sciences/statistics/1726227>. Acesso em 23 de fevereiro de 2013.

SOARES, V. C. A. Aplicações do problema de otimização de carteiras de investimento. 2011. 72 f. Dissertação (Mestrado em Matemática) – Instituto de Matemática, Estatística e Computação Científica, Universidade Estadual de Campinas, São Paulo. 2011

VIEIRA, S. P.; MENDES, A. G. S. T. Governança corporativa: uma análise de sua evolução e impactos no mercado de capitais brasileiro. **Revista do BNDES**, Rio de Janeiro, v. 11, n. 22, p. 103-122, 2013.

ZENIOS, S.A.; KANG, P., Mean-absolute deviation portfolio optimization for mortgage-backed securities. **Annals of Operations Research**, v. 45, p. 433-450, 1993.

APPENDIX I – Percentual invested by the optimized portfolio for each period

Table I.1 – Investment of the optimized portfolio for each period (2007-2009)

Companies	2007 1º Q	2007 2º Q	2007 3º Q	2007 4º Q	2008 1º Q	2008 2º Q	2008 3º Q	2008 4º Q	2009 1º Q	2009 2º Q	2009 3º Q	2009 4º Q
BRADESCO	-	-	0,09	0,13	-	0,10	-	-	-	0,18	-	-
BRASIL	0,41	0,02	-	-	-	-	-	-	0,06	-	-	-
BRASKEM	-	-	-	-	0,01	0,01	0,01	0,01	0,01	0,14	0,01	0,01
BRF FOODS	-	-	-	-	-	-	-	-	0,04	0,04	0,04	0,04
CCR RODOVIAS	0,05	0,05	0,05	0,05	-	-	-	-	-	-	-	-
CEMIG	-	-	-	-	-	0,18	-	-	-	-	-	-
COPEL	-	-	-	-	-	-	-	-	-	0,06	0,06	0,06
CPFL ENERGIA	-	-	-	-	0,18	-	-	-	-	-	-	-
DURATEX	-	-	-	-	-	-	-	-	0,06	-	-	0,05
ELETROBRÁS	-	-	-	-	-	-	0,08	0,08	-	-	-	-
ELETROPAULO	-	0,33	0,09	-	-	-	-	-	0,49	-	-	-
EMBRAER	-	-	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
EVEN	-	-	-	-	-	-	-	-	0,03	0,07	0,05	0,03
GERDAU	0,12	-	0,12	-	0,11	0,43	0,11	0,11	-	0,19	-	0,13
GERDAU MET	-	0,12	-	0,12	-	-	-	-	-	-	-	-
IOCHP-MAXION	0,08	0,12	-	-	-	-	-	-	-	-	-	-
ITAUBANCO	-	0,11	0,04	-	0,10	-	0,10	0,10	-	-	-	-
ITAUSA	-	-	-	-	-	-	-	-	0,14	-	0,20	0,15
LIGHT S/A	-	-	-	-	-	-	0,01	0,01	-	-	-	-
NATURA	0,03	0,03	0,03	0,03	0,47	0,05	0,59	0,59	0,04	0,04	0,30	0,04
ODONTOPREV	-	-	-	-	-	0,07	0,07	0,07	-	-	-	-
PETROBRÁS	0,17	0,17	0,17	0,29	-	-	-	-	-	-	-	-
SABESP	0,05	0,05	-	-	0,07	-	-	-	-	-	-	-
SULAMERICA	-	-	-	-	-	-	-	-	0,11	-	0,18	0,41
SUZANO PAPEL	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	-	0,20	-	-
TRACTEBEL	0,09	-	-	0,14	0,03	-	-	-	0,01	0,01	0,01	0,01
USIMINAS	-	-	-	-	-	-	-	-	-	-	0,13	-

WEG	-	-	0,35	0,07	-	-	-	-	-	-	-	-

Source: own elaboration

Table I.2 – Investment of the optimized portfolio in each period (2010-2012)

Companies	2010 1º Q	2010 2º Q	2010 3º Q	2010 4º Q	2011 1º Q	2011 2º Q	2011 3º Q	2011 4º Q	2012 1º Q	2012 2º Q	2012 3º Q
BRADESCO	0,016	0,016	-	-	-	-	-	-	-	-	-
BRASKEM	0,011	0,011	0,011	0,011	0,014	0,014	0,014	0,014	0,014	0,014	0,014
BRF FOODS	0,047	0,047	0,047	0,047	0,048	0,048	0,048	0,252	0,048	0,048	0,048
CEMIG	-	-	-	-	-	-	-	-	0,047	0,047	0,047
COPASA	-	-	-	-	-	-	0,252	0,092	-	-	-
COPEL	0,130	0,130	-	-	0,172	0,172	-	-	-	-	-
DURATEX	-	-	-	-	0,083	0,083	-	-	0,009	0,009	-
ECORODOVIAS	-	-	-	-	0,029	0,029	0,029	0,029	0,029	0,029	0,029
EMBRAER	0,009	0,009	0,009	0,021	0,005	0,005	0,005	0,005	-	-	-
ENERGIAS BR	-	-	-	-	-	-	-	0,047	-	-	-
EVEN	0,067	0,067	0,102	0,543	0,174	0,174	0,109	0,109	0,109	0,109	0,109
FIBRIA	-	-	-	-	0,013	0,013	-	-	-	-	-
GERDAU	0,096	0,096	0,096	0,096	-	-	-	-	-	-	0,082
GERDAU MET	-	-	-	-	0,079	0,079	0,079	0,079	0,082	0,082	-
ITAUSA	0,067	0,067	0,068	0,073	0,009	0,009	-	-	0,007	0,007	-
LIGHT S/A	-	-	-	-	-	-	0,047	-	-	-	-
NATURA	0,409	0,409	0,503	0,044	0,048	0,048	0,048	0,048	0,048	0,048	0,253
SABESP	-	-	-	-	-	-	-	-	0,281	0,281	0,092
SULAMERICA	0,125	0,125	0,023	-	0,153	0,153	0,153	-	0,153	0,153	0,153
TIM PARTC.S/A	-	-	-	-	0,042	0,042	0,071	0,028	0,028	0,028	0,028
TRACTEBEL	0,023	0,023	0,023	0,023	-	-	0,013	0,013	0,013	0,013	0,013
VALE	-	-	-	-	0,133	0,133	0,133	0,133	0,133	0,133	0,133

Source: own elaboration